Introduction to Calculus: Limits, Derivatives, and Integrals

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Overview:
Calculus is a branch of mathematics that studies how things change and accumulative quantities. It provides powerful tools for analyzing functions, modeling real-world phenomena, and solving problems in science and engineering.

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1. Limits:
- Definition: The limit of a function $f(x)$ as x approaches a value c is the value that $f(x)$ gets closer to as x approaches c.

- Notation:
$$\lim_{x \to c} f(x) = L$$

- Importance: Limits help us understand the behavior of functions at points where they may not be explicitly defined, and serve as the foundation for derivatives and integrals.

- Example:
$$\lim_{x \to 2} (x^2 - 4)/(x - 2) = 4$$

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2. Derivatives:

- Definition: The derivative of a function $f(x)$ at a point x measures the instantaneous rate of change or the slope of the tangent line at that point.

- Formal Definition (Limit form):
  $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Interpretation:
  - Represents how a quantity changes over an infinitesimal interval.
  - Used in optimization, motion analysis, and curve sketching.

- Common Rules:
  - Power rule: $\frac{d}{dx} x^n = n x^{n-1}$
  - Sum rule: $(f+g)' = f' + g'$
  - Product and quotient rules

- Example:
  $$\frac{d}{dx} (3x^2 + 2x) = 6x + 2$$

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3. Integrals:

- Definition: Integration is the process of finding the accumulated quantity, such as area under a curve.

- Indefinite Integral:
  $$\int f(x) \, dx$$
  Represents a family of functions whose derivatives are $f(x)$.

- Definite Integral:
  $$\int_a^b f(x) \, dx$$
  Calculates the net area under $f(x)$ between a and b.

- Fundamental Theorem of Calculus:
  Connects differentiation and integration:

$$ \frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x) $$

- Example:
  $$ \int x^2 dx = \frac{x^3}{3} + C $$

Summary:
- Limits help analyze function behavior around points.
- Derivatives quantify instantaneous change.
- Integrals accumulate quantities like area and total change.

Mastering these concepts provides a solid foundation for advanced mathematics, physics, and engineering applications.
